LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIFTH SEMESTER – **NOVEMBER 2023**

UMT 5501 – REAL ANALYSIS - II

Date: 30-10-2023 Dept. No. Max. : 100 Marks

Time: 09:00 AM - 12:00 NOON

PART – A

Answer ALL the questions: **(10 × 2 = 20)**

1. Define limit of a function.
2. Show that $\lim_{x\to 0} sgn(x)$ does not exist in $\mathbb{R}$, where sgn is the signum function defined by
   $$sgn(x) = \begin{cases} 1 \ \ if\ x > 0 \\ 0 \ \ if\ x = 0 \\ -1 \ \ if\ x < 0 \end{cases}.$$
3. State divergence criteria.
4. Give an example each for a continuous and a discontinuous function.
5. Write the non-uniformity criterion.
6. State interior extremum theorem.
7. List any two properties of an open set.
8. Define Riemann sum.
9. What are the properties of Riemann integrable functions?
10. State preservation of compactness theorem.

PART – B

Answer any FIVE of the following: **(5 × 8 = 40)**

11. State and prove sequential criterion for limits.
12. State and prove Bolzano's intermediate value theorem.
13. Let $A \subseteq \mathbb{R}$, let $f: A \to \mathbb{R}$ and let $|f|$ be defined by $|f|(x) = |f(x)|\ \forall\ x \in A$. Prove that $|f|$ is continuous at a point c if f is continuous at c.
14. State and prove Rolle's theorem.
15. State and prove Cauchy's criterion.
16. Let $f: [a, b] \to \mathbb{R}$ be monotone on $[a, b]$. Prove that $f \in \mathcal{R}[a, b]$.
17. State and prove fundamental theorem of calculus.
18. Give a characterization of a closed set. Validate the statement.

PART – C

Answer any TWO of the following: **(2 × 20 = 40)**

19. a) State and prove maximum - minimum theorem.

    b) State and prove squeeze theorem for continuous functions.

    (12 + 8)

20. State and prove Taylor's theorem.

    (20)

21. a) State and prove Cauchy's mean value theorem.

    b) State and prove boundedness theorem for a Riemann integrable function.

    (10 + 10)

22. a) State and prove Heine-Borel theorem.

    b) State and prove preservation of compactness theorem.

    (12 + 8)

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